Acyclicity Conditions and their Application to Query Answering in Description Logics

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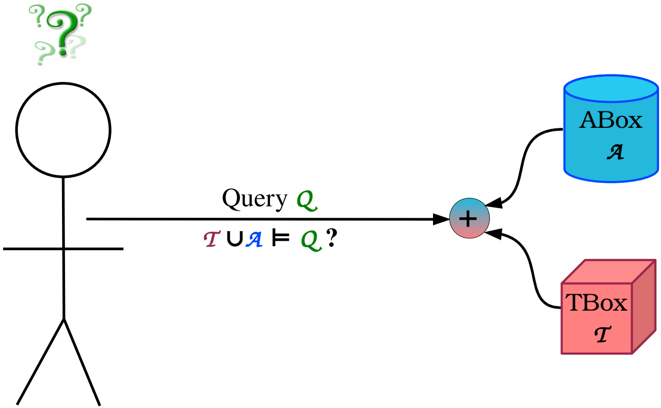
Outline

ONTOLOGICAL QUERY ANSWERING

- Key reasoning task for DL and rule-based applications



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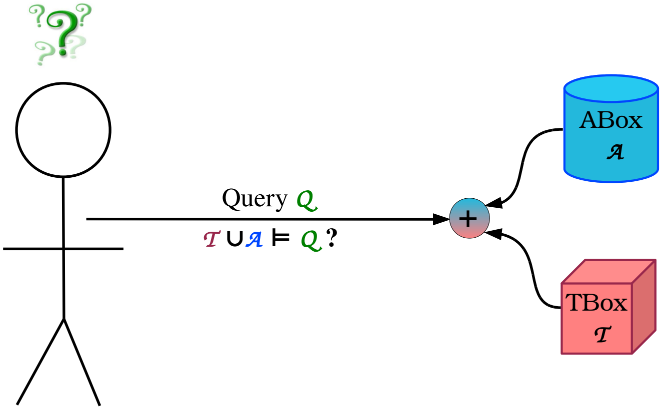

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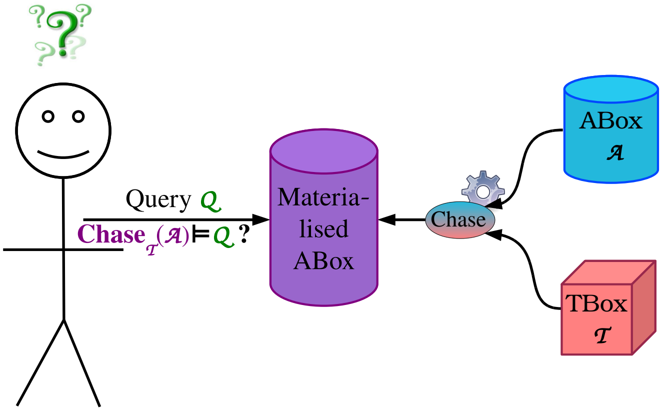
- Materialisation-based paradigm: chase ABox using TBox and evaluate Q in the computed ABox



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  1. Schema constraints in databases
  2. Data transformation rules in data exchange
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II. Acyclicity conditions might be too restrictive

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- Answering CQs over expressive DLs is expensive, e.g. EXPTIME-complete for Horn-$\mathcal{SHOIQ}$ [Ortiz, Rudolph and Simkus, 2011]

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- Suggestion: materialise ABoxes only over acyclic TBoxes
  - Always complete ✓
  - Provably terminating ✓

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Materialisation-based reasoning beyond OWL 2 RL
might be practically feasible

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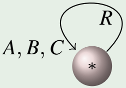

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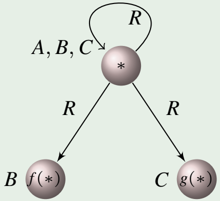

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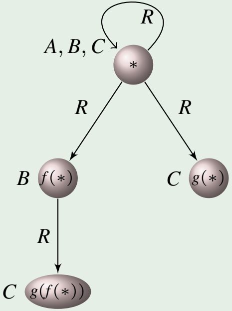

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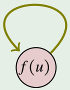

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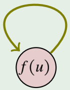

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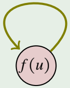

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- May overestimate rule applicability

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- Track rule applications more 'faithfully'

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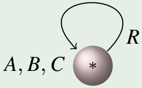

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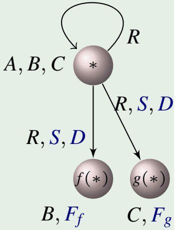

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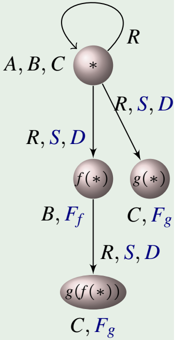

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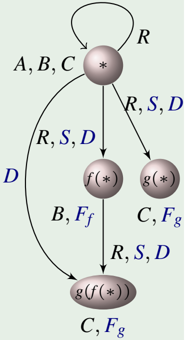

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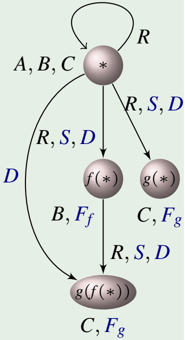

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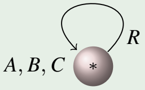

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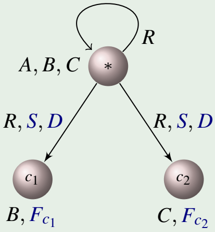

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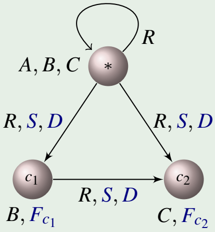

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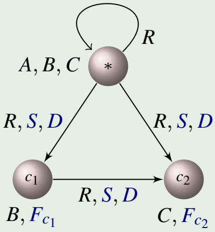

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- Horn-$\mathcal{SHIQ}$ TBoxes can be checked in PTIME for MSA before *potential* materialisation-based query answering

ACYCLICITY CONDITIONS (PARTIAL) TAXONOMY

JA $\subsetneq$ SWA MSA MFA

Acyclicity Conditions (Partial) Taxonomy

- Our contributions:

JA	$\subsetneq$	SWA		MSA	MFA

Acyclicity Conditions (Partial) Taxonomy

- Our contributions:
  1. MSA strictly subsumes SWA

$$\text{JA} \subsetneq \text{SWA} \subsetneq \text{MSA} \qquad \text{MFA}$$

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MFA but not MSA

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MFA but not MSA

- MSA and MFA coincide in experimental evaluation of DL ontologies

Outline

1 Motivation

2 MFA and MSA

3 Querying Acyclic DL Ontologies

4 Experimental Results

Translating DLs into rules

- Axioms of normalised Horn-$\mathcal{SRIQ}$ ontologies can be converted to (existential) rules

$A \sqsubseteq \exists R.B$	$A(x) \to \exists y.R(x,y) \land B(y)$
$A \sqsubseteq \leq 1\, R.B$	$A(z) \land R(z,x_1) \land B(x_1) \land R(z,x_2)$
	$\land\ B(x_2) \to x_1 \approx x_2$
$A \sqcap B \sqsubseteq C$	$A(x) \land B(x) \to C(x)$
$A \sqsubseteq \forall R.B$	$A(z) \land R(z,x) \to B(x)$
$R \sqsubseteq S$	$R(x_1,x_2) \to S(x_1,x_2)$
$R \circ S \sqsubseteq T$	$R(x_1,z) \land S(z,x_2) \to T(x_1,x_2)$

TRANSLATING DLS INTO RULES

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$A \sqsubseteq \exists R.B$	$A(x) \to \exists y.R(x,y) \wedge B(y)$
$A \sqsubseteq \leq 1\, R.B$	$A(z) \wedge R(z,x_1) \wedge B(x_1) \wedge R(z,x_2)$
	$\wedge B(x_2) \to \underline{x_1 \approx x_2}$
$A \sqcap B \sqsubseteq C$	$A(x) \wedge B(x) \to C(x)$
$A \sqsubseteq \forall R.B$	$A(z) \wedge R(z,x) \to B(x)$
$R \sqsubseteq S$	$R(x_1,x_2) \to S(x_1,x_2)$
$R \circ S \sqsubseteq T$	$R(x_1,z) \wedge S(z,x_2) \to T(x_1,x_2)$

- Equality is handled with a modification of the singularisation [Marnette, PODS, 2009] technique

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2. Horn-$\mathcal{SRI}$ TBox $\mathcal{T}$ and ABox $\mathcal{A}$
   $\mathcal{T}$ is weakly acyclic
   F set of facts

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Outline

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1K–5K	20	14	14	12
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- 7 large and expressive OBO ontologies MSA but not JA (only two of them were $\mathcal{ELH}^r$ and DL-Lite)

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Depth	#	generated size		materialisation size	
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10–80	14	281	51	283	53

$$\text{Depth} = \text{length of function symbol nesting}$$

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SUMMARY OF THE RESULTS

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2. Complexity analysis for checking MSA and MFA

	Horn-$\mathcal{SHIQ}$	bounded arity	no restriction
MSA	PTime-complete	coNP-complete	ExpTime-complete
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   - Horn-$\mathcal{SHIQ}$ $\mathcal{T}$ in MFA: $\mathcal{T} \cup \mathcal{A} \models Q$ is PSpace-complete
4. Experimental evaluation on DL ontologies
   - 83% ontologies found acyclic (78% JA)
   - materialised ABoxes not too large $\rightsquigarrow$ $\times$ 5 bigger on average for ontologies with depth < 5 (= most ontologies)

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Thank you! Questions?!?